SPEED OF SOUND AND SPEED OF LIGHT IN AIR

1. Introduction

Sound can be determined as a longitudinal, mechanical wave motion and its propagation through the medium can be described either by studying displacements of the medium particles or by considering varying of pressure in time caused by the sound wave. Because sound is longitudinal wave motion the displacements of the medium particles around their equilibrium positions take place in the propagation direction of sound. As all mechanical waves, sound needs a medium, which can be either gas, fluid or solid for propagation. The propagation speed of sound depends on the medium. In this exercise you measure the speed of sound in air, which can be considered as an ideal gas in the measurement conditions.

In the second part of this exercise you measure the speed of light in air. The speed of light in vacuum, usually denoted with the symbol \( c \), is an important universal physical constant and its value is nowadays fixed to the value \( c = 299792458 \) m/s. The length of the meter is defined from the speed of light and from the international standard for time. The speed of light \( v \) in transparent materials is less than \( c \). The ratio between \( c \) and the speed of light in a medium is called the refractive index \( n \) of the material, i.e. \( n = c / v \). For example, for the visible light used in these measurements the refractive index of air is around 1.0003, so the speed of light measured here is near \( c \).

2. Speed of sound in air

2.1 Exercises

Solve the following exercises before coming to the laboratory. There is a form at the end of this instruction (Attachment 1) for the answers of the exercises 1 and 2. Return the form to the tutor.

1. Define the following concepts: a) Wavelength, b) frequency and c) phase difference between two waves.
2. List phenomena in everyday life, in which it can be observed that the speed of sound has a finite value.
3. Plan a measurement form for your observations. For the planning read carefully chapter 2.5 describing the measurements. You can fulfil your form with Excel either before coming to the laboratory or in the beginning of the measurements and register your observations directly to this electrical form.
2.2 Speed of sound in an ideal gas

The propagation speed $v$ of a periodic longitudinal or transverse wave motion can always be expressed as the product of the wavelength $\lambda$ and frequency $f$ of the wave, i.e.

$$v = \lambda f.$$  \hspace{1cm} (1)

In this exercise you determine the speed of sound in air with the aid of Eq. (1) by measuring the frequencies of the sound waves with a frequency counter and by determining the wavelengths with methods described in chapter 2.5.

When the sound propagates in air at room temperature and room pressure air can be assumed to behave like an ideal gas. When the longitudinal mechanical wave propagates in a fluid or in a gas, the propagation speed can be calculated with the aid of bulk modulus $B$ and the density $\rho$ of the gas from an equation

$$v = \sqrt{\frac{B}{\rho}}.$$ \hspace{1cm} (2)

The bulk modulus is defined with pressure $p$ and volume $V$ as follows

$$B = -V \frac{dp}{dV}.$$ \hspace{1cm} (3)

If the propagation of the sound in the air is an adiabatic process, where no heat change takes place during the compression and the expansion, the expression between pressure and volume is of the form

$$pV^\gamma = \text{constant},$$ \hspace{1cm} (4)

where $\gamma = C_p/C_v$ is the ratio of the constant pressure and the constant volume heat capacities. By differentiating Eq. (4) with respect to the volume we get

$$\frac{dp}{dV}V^\gamma + \gamma pV^{\gamma-1} = 0.$$ 

Then the derivative of the pressure with respect to the volume takes the form

$$\frac{dp}{dV} = -\frac{\gamma pV^{\gamma-1}}{V^\gamma} = -\frac{\rho}{V}.$$ 

Using this together with Eq. (3) we get for the bulk modulus

$$B = -V\left(-\frac{\rho}{V}\right) = \gamma p.$$ 

Thus, the speed of sound is according to Eq. (2)

$$v = \sqrt{\frac{\gamma p}{\rho}}.$$ \hspace{1cm} (5)

With the aid of the equation of state for ideal gas $pV = nRT$ the density $\rho$ of the gas can be expressed as
\[ nRT = \frac{m}{M}RT = \frac{\rho V}{M}RT = pV \Rightarrow \rho = \frac{pM}{RT}, \]

where \( n \), \( m \) and \( M \) are the amount of the gas in moles, the mass and the molar mass of the gas, respectively, \( R = 8.314 \text{JK}^{-1}\text{mol}^{-1} \) is the gas constant and \( T \) is the absolute temperature. So, the speed of sound in the ideal gas can be expressed as follows:

\[ v = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{pRT}{pM}} = \sqrt{\frac{\gamma RT}{M}}. \quad (6) \]

By using Eq. (6) a reference value for the speed of sound can be calculated when the room temperature is measured. The average molecular mass of air is \( M = 29.0 \text{g/mol} \) and air is two atomic gas for which \( \gamma = 7/5 \).

### 2.3 Measurement arrangement

The diagram representing the measurement arrangement of the sound speed is shown in Fig. 1 and the measurement equipment is seen in Fig. 2. A sinusoidal voltage from a signal generator is connected to the Y-plates of an oscilloscope and to a frequency counter. The same voltage also drives a speaker, which changes the voltage to a sinusoidal sound wave, which has the same frequency as the voltage. This sound propagates in air to a microphone, which changes it again to a sinusoidal voltage. The voltage from the microphone is connected to the X-plates of the oscilloscope. Because sound propagates from the speaker to the microphone at a finite speed the voltages from the microphone and straight from the generator are usually out of phase.

![Figure 1 Measurement arrangement of the sound speed.](image)
The phase difference between the two voltages depends on the position of the microphone with respect to the speaker. During the measurements the distance between the speaker and the microphone is changed by moving the microphone on the rails. Now the voltage straight from the generator deflects the point on the oscilloscope’s screen horizontally and the voltage from the microphone vertically. In the next chapter 2.4 we investigate what kind of figures, the so called Lissajous figures, are seen on the screen when the phase difference varies.

2.4 Lissajous figures

If a point is at the same time involved in two periodic wave motions with the same angular frequency $\omega$ taking place perpendicular to one another the place of the point in the $(x, y)$– coordinate system as a function of time $t$ can be expressed as

$$\begin{align*}
  x &= A \sin(\omega t) \\
  y &= B \sin(\omega t + \theta)
\end{align*}$$

(7)

where $A$ and $B$ are the amplitudes of the two wave motions and $\theta$ is the phase difference between the waves. When the term $\sin(\omega t)$ calculated from the first equation above is set on the second one we get
which in general case represents an ellipse, whose center is at the origin. Depending on the phase difference various figures take place:

a) When the waves are in phase, i.e. when $\theta = 0$, Eq. (8) gives an equation of the line

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 - 2 \frac{xy}{AB} \cos \theta = \sin^2 \theta,$$  \hspace{1cm} (8)

$$= \frac{2}{AB} \left(\frac{x - y}{B}\right)^2 = 0 \Rightarrow y = \frac{B}{A} x.$$  \hspace{1cm} (9)

b) When the phase difference is $\theta = 90^\circ$, Eq. (8) gives

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 = 1,$$  \hspace{1cm} (10)

which is an equation of an ellipse, whose axes coincide with the coordinate axes. If the amplitudes of the waves are equal, i.e. if $A = B$ we get an equation of a circle.

c) If the phase difference between the waves is half wave, i.e. if $\theta = 180^\circ$, Eq. (8) gives a line equation of the form

$$y = -\frac{B}{A} x.$$  \hspace{1cm} (11)

### 2.5 Measurement and analysis

Before starting the measurements fulfill your measurement form with Excel. Use Excel file also for the calculations needed for the analysis of your results.

First, study the measurement equipment and the use of the oscilloscope with the tutor. Adjust the distance between the microphone and the speaker and observe what kind of figures can be seen on the oscilloscope’s screen. What are the corresponding phase differences? Register your observations to the form given in Attachment 1.

Set the frequency of the voltage from the signal generator around 6.0 kHz and register the value of the frequency $f_1$ from the counter to your measurement form. Move the microphone close to the speaker at a position, where the phase difference is according to the oscilloscope zero (or $180^\circ$). Register the place of the microphone $s_0$ from the metric scale below the rails. Then move the microphone further away from the speaker to the position $s_1 = s_0 + \lambda$, where the phase difference is again zero (or $180^\circ$). Continue moving the microphone until it is at the position $s_k = s_0 + k\lambda$, where $k = 8 - 10$. Register the positions $s_0$ and $s_k$ as well as the number of wave lengths ($k$) to your
measurement form. Calculate and register the corresponding wavelength \( \lambda_1 \) and find the value of the sound speed \( v_1 \) using Eq. (1).

Then increase the frequency around the value \( f_2 = 6.2 \) kHz and repeat the measurements to get the value \( v_2 \) for the speed of sound. Continue this procedure by increasing the frequency at about 0.2 kHz steps between the observations until you have measured the sound speed \( v_i \) with 10 frequency values between 6 – 8 kHz. Calculate the mean value \( \bar{v} \) of the observations for the final result.

The error limit of the mean value is supposed to be its standard error \( \sigma \) and therefore the deviations of the observations \( (v_i - \bar{v}) \) as well as their squares \( (v_i - \bar{v})^2 \) are calculated. List the values of these deviations and their squares to your measurement form. The expression of the standard error \( \sigma \) of the mean value is of the form

\[
\sigma = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (v_i - \bar{v})^2},
\]

where \( n \) is the number of the observations. Calculate also the sum of the squares \( \sum_{i=1}^{n} (v_i - \bar{v})^2 \) and value of the standard error \( \sigma \) with Excel.

For calculating the reference value for the speed of sound from the ideal gas Eq. (6) measure the room temperature with a thermometer seen in Fig. 2. Register the temperature value as well as the calculated speed of sound to your measurement form. Notice that the temperature in Eq. (6) is the absolute temperature.

2.6 Results and discussion

The report from the sound speed measurement is your measurement form in which all your observations and calculations are seen. The form should include all the calculated sound speeds as well as their mean value and the differences, squares and the sum calculated for the error evaluation. Give both the mean value \( \bar{v} \) of the sound speed with its error limits and the ideal gas reference value as results. Discuss the following questions and list your conclusions to the attached returnable form.

What reasons can cause the possible deviation between the measured sound speed and the reference value?

Why were the frequencies between 6-8 kHz used in the measurements? What would happen if the frequencies were higher/lower?
3. Speed of light in air

The speed of light was measured by several means before its value was fixed. For example, a Danish astronomer Ole Rømer measured in 1675 the periods of Jupiter’s moon Io (about 42.5 h) and observed that the measured periods were shorter as Earth approached Jupiter and longer as Earth moved away. From this observed retardation of light Rømer concluded that light has a final speed. A Dutch scientist Christiaan Huygens used Rømer’s observations and calculated that the speed of light was $2.25 \times 10^8$ m/s. A French physicist Armand Hippolyte Louis Fizeau measured in 1849 the speed of light using a light beam reflected from a mirror eight kilometers away. The light beam to the mirror passed through a gap of a rapidly rotating teeth wheel, whose rotation rate was adjusted so that the light returning from the mirror passed the next gap. When the distance between the wheel and the mirror, the number of teeth on the wheel and the rotation rate were known, Fizeau calculated that the speed of light was $3.133 \times 10^8$ m/s.

3.1 Measurement arrangement

In this exercise the speed of light in air is measured using very short light pulses. The measurement arrangement is shown schematically in Fig. 3. The light pulse from the source is split with a beam splitter into two beams; a reference beam and a measurement beam. Both the beams are reflected back to the beam splitter from the reflectors and they are observed with the same detector. If the travel distance of the measurement beam differs from that of the reference beam, the two beams reach the detector at the different time. When the path difference $\Delta s$ and the time difference $\Delta t$ between the beams are measured the speed of light in air can be calculated from equation

$$v = \frac{\Delta t}{\Delta t}.$$

![Diagram of measurement arrangement of the light speed.](image)

**Figure 3** Measurement arrangement of the light speed.
The structure of the measurement equipment is shown in Fig. 4 and a photograph of the equipment during the measurement is seen in Fig. 5. A diode used as a light source, a beam splitter and a photo diode detector are placed in a box. On the top of the box there is an aperture from which the reference beam gets to the reflector placed onto the aperture.

![Figure 4 Structure of the measurement equipment of the light speed.](image)

Diode (a) emits short light pulses at a frequency of 40 kHz. These pulses hit on the beam splitter (b), which reflects a part of the beam through the aperture (c₁) to the reflector (d₁). This beam is called the reference beam and it reflects back through the beam splitter to the detector diode (e). The detector pulse is connected through a PULSES interface to an oscilloscope, where it is seen as a first pulse on a screen.

Another part of the beam goes through the beam splitter to the aperture (c₂) at the front side of the box. This measurement beam propagates through the lens (f) to the reflector (d₂). The lens (f) is used to converge the strongly diverging beam from the diode and to focus it to the reflector d₂. The measurement beam returns along the same path to the beam splitter, hits to the detector diode and it is seen as a second pulse on the oscilloscope’s screen. The distances c₁ – b and c₂ – b from the beam splitter to the apertures are the same and so the time distance between the pulses results merely from the distance d₂ – c₂ travelled back and forth by the measurement beam.
3.2 Exercises

Solve the following exercises before coming to the laboratory. Use the form in Attachment 1 for the answers of the exercise 4.

4. a) What is the time difference between the measurement and the reference beam, when the measurement equipment is set at a distance of 10 m from the reflector $d_2$? Use the value 299792458 m/s for the speed of light.

b) What should the distance between the reflector $d_2$ and the equipment be so that the time difference would be 1s?

5. Plan a measurement form for your results. For the planning read chapter 3.3 describing the measurements. You can fulfil your form with Excel and use it when analyzing the results.

3.3 Measurements and analysis

In order to get a suitable time difference between the reference and the measurement beam the distance from the box to the reflector $d_2$ should be long enough. Thus the measurements have to be carried out in a corridor of the laboratory.

The measurement equipment is placed on a movable table. Transfer the table to the corridor and place the reflector $d_2$ to a stand attached to a balk. Set the table to a distance of about 10 m from the reflector. Adjust the equipment, e.g. the positions of the
table, the reflector $d_1$ and the lens so that the pulses on the screen have the same amplitude. Measure the time difference $\Delta t$ between the pulses with the oscilloscope and the distance $s/2$, which is a half of a path difference between the beams, with a measuring tape. Repeat the measurements five times by moving the table about one to two meters between the observations.

The speed of light can be calculated in each measurement point from Eq. (13) as a ratio between the path difference and the time difference. The final result is the mean value of these five ratios. For the error evaluation calculate the maximum deviation of the mean value. The other means is to replace the five points to $(t, s)$–coordinate system and to fit a straight line to the points. The speed of light can then be determined as a slope of the line. These calculations can be performed using the LINEST-function available in Excel.

### 3.4 Results

The report from the light speed measurement is your measurement form in which all your observations and calculations are seen. The form should include the calculated speeds of light as well as their mean value and the graph presented in $(t, s)$–coordinate system with the information from the fit. Give both the mean value and the slope of the line with their error limits as results.
Attachment 1: Form for the exercises

1. Define the following concepts: a) Wavelength, b) frequency ja c) phase difference between two waves.
   Wavelength:
   ________________________________________________________________
   ________________________________________________________________
   Frequency:
   ________________________________________________________________
   ________________________________________________________________
   Phase difference between two waves:
   ________________________________________________________________
   ________________________________________________________________

2. List phenomena in everyday life, in which it can be observed that the speed of sound has a finite value.
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________

4. a) What is the time difference between the measurement and the reference beam, when the measurement equipment is set at a distance of 10 m from the reflector $d_2$? Use the value 299792458 m/s for the speed of light.
   
   b) What should the distance between the reflector $d_2$ and the equipment be so that the time difference would be 1s?
Observations of the figures on the oscilloscope’s screen in the speed of sound measurement:

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

Conclusions from the speed of sound measurement:

What reasons can cause the possible deviation between the measured sound speed and the reference value?

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Why were the frequencies between 6-8 kHz used in the measurements? What would happen if the frequencies were higher/lower?

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